

Numerical Investigation for Convective Heat Transfer of CNT Nano-Fluids over a Stretching Surface

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Abstract. The performance of carbon nanotube (CNT) nanofluids on convective heat transfer over a stretching sheet was investigated under thermal stratification and magnetic field effects. Water, engine oil and ethylene glycol are used as the base fluids. The governing equations are transformed into a system of coupled nonlinear ordinary differential equations using similarity transformations and solved numerically using the fourth-order Runge–Kutta–Fehlberg in conjunction to shooting method. The CNT nanofluids with an engine oil base fluid shows the highest thermal conductivity in comparison to ethylene glycol and water, respectively. Potential application of the thermal conductivity enhancement of CNT nanofluid is to increase the energy-efficient mechanical systems in heating, cooling and ventilation of the indoor environment.

Introduction

The thermal conductivity of fluid heat transfer plays an important role in the development of energy-efficient mechanical systems used in heating, cooling and ventilation of the indoor environment. Choi [1] who was first introduce a concept of nanofluid as a new approach to enhance the thermal conductivity of the base fluids. Nanofluid is a colloidal suspension of nanoparticles in the range of size 1 – 100 nm in the conventional base fluid such as water, ethylene glycol and engine oil. The various engineering applications of nanofluids due to the enhancement of thermophysical properties is reported by [2-3].

CNT nanofluids is type of nanofluids with high thermal conductivity due to the nano-sized particles carbon nanotubes. CNTs can be categorized as either single-walled (SWCNT) or multi-walled nanotubes (MWCNT). Further studies on CNT have been conducted by many researchers to reveal the thermo-physical properties. Philip and Shima [4,5] reported the heat transfer enhancement of CNT nanofluid due to the volume fraction. Ding et al. [6] investigated that thermal conductivity of CNT based nanofluid increases significantly with the temperature at the same volume fraction. Nanofluids with low concentration of Cu, CuO, or CNTs have considerably higher thermal conductivity than identical base liquids as reported by Min et al. [7]. Enhancement of heat transfer rate due to CNT nanofluids on convective heat transfer over various surfaces and geometry are already reported by many investigators such as [8 – 11]. In addition, MHD flows of CNT nanofluids are also investigated by [12 – 14] numerically or experimentally due to some engineering applications. Influence of thermal stratification is an important aspect in heat transfer analysis and applications. Effects of thermal stratification in convective flow past a surface was studied by [15].

Based on the best of the authors knowledge, convective heat transfer of nanofluids over stretching surface has not received much attention. The objective of the present work is to analyze the performance of the SWCNTs and MWCNTs nanofluids (with base fluids are water, engine oil and ethylene glycol) flow and heat transfer over a stretching sheet with slip condition. Moreover, we also considered the viscous dissipation, thermal stratification and magnetic field effects.

Mathematical Analysis

Consider two - dimensional steady boundary layer MHD flow and heat transfer of CNT nanofluids over a stretching sheet embedded in a thermally stratified medium. It is considered that the sheet is stretched along the x - axis . (see Fig. 1).

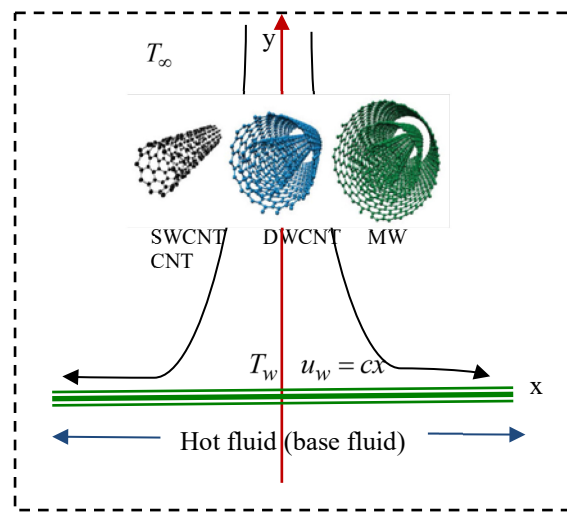


Figure1: Physical model and coordinate system.

Moreover, fluid is placed on the half plane $y > 0$. It is also considered that the sheet is stretched with the velocity $u_w = cx$, $c > 0$. The hot base fluid without incorporating the nanotubes effects is also fixed along with the lower surface of the wall in such a way that temperature of the hot fluid, T_w is greater than the ambient fluid temperature, $T_\infty = T_{\infty,0} + Ax$, where A is constants and varied to alter the intensity of stratification in the medium, $T_{\infty,0}$ is ambient temperature. We considered a uniform magnetic field B_0 applied parallel to y -axis, while the induced magnetic field is assumed to be negligible. The base fluids and the CNTs are assumed to be in thermal equilibrium. The viscous dissipation effect is also considered in the energy equation. The term $Q_0(T_\infty - T)$ is assumed to be the amount of heat generated/absorbed per unit volume. Q_0 is a constant, which may take on either positive or negative values. When the wall temperature T_w exceeds the free stream temperature T_∞ , the source term represents the heat source when $Q_0 < 0$ and heat sink when $Q_0 > 0$.

Using the standard boundary-layer approximation, the governing equations for the continuity, momentum and energy in boundary layer flow can be written as,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{nf} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_{nf}} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} - \frac{\nu_{nf}}{(\rho c_p)_{nf}} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{Q_0 (T - T_\infty)}{(\rho c_p)_{nf}} \quad (3)$$

where u and v are the velocity components in the x and y directions. Symbols T , ρ_{nf} , μ_{nf} , α_{nf} are temperature, effective density and effective dynamic viscosity and thermal diffusivity of the nanofluid, respectively, which are defined as

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_{CNT}, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}},$$

$$(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_{CNT}, \quad \frac{k_{nf}}{k_f} = \left\{ \frac{(k_{CNT} + 2k_f) - 2\phi(k_f - k_{CNT})}{(k_{CNT} + 2k_f) + 2\phi(k_f - k_{CNT})} \right\} \quad (4)$$

In the limit of low particle volume concentration (ϕ) and the nanoparticle conductivity (k_s), being much higher than the base liquid conductivity (k_f), where μ_f is the viscosity of base fluid, ϕ is the nanoparticle fraction, $(\rho c_p)_{nf}$ is the effective heat capacity of a CNT, k_{nf} is the thermal conductivity of nanofluid, k_f and k_{CNT} are the thermal conductivities of the base fluid and CNT, while ρ_f and ρ_{CNT} are the densities of the base fluid and CNT, respectively. The corresponding boundary conditions are

$$u = u_w + \gamma \frac{\partial u}{\partial y}, \quad v = 0, \quad T = T_w \text{ at } y = 0; \quad u \rightarrow 0, v \rightarrow 0, \quad T \rightarrow T_\infty = T_{\infty,0} + Ax \text{ as } y \rightarrow \infty \quad (5)$$

where $T_w - T_{\infty,0} = nx$, $u_w = cx$ and m, n, c and A are constants and γ is the slip parameter.

Introducing the following similarity transformations,

$$\eta = \sqrt{\frac{c}{\nu_f}} y, \quad \psi = \sqrt{c\nu_f} x f(\eta), \quad \theta(\eta) = \frac{T - T_{\infty,0}}{T_w - T_{\infty,0}} - \frac{Ax}{T_w - T_{\infty,0}} \quad (6)$$

The continuity equation is satisfied if we choose a stream function ψ defined as

$$u = \partial\psi / \partial y, \quad v = -\partial\psi / \partial x \quad (7)$$

Equations (1–3) are transformed into the non-dimensional form as follows:

$$f''' + (1 - \phi)^{2.5} \left[\left(1 - \phi + \phi \frac{\rho_{CNT}}{\rho_f} \right) (ff'' - f'^2) - Mf' \right] = 0 \quad (8)$$

$$\frac{1}{\text{Pr}} \frac{k_{nf}}{k_f} \theta'' - \frac{Ec}{(1 - \phi)^{2.5}} (f'')^2 - \delta \theta - \left(1 - \phi + \phi \frac{(\rho c_p)_{CNT}}{(\rho c_p)_f} \right) \left[f'\theta - \frac{\varepsilon}{4} f' - f\theta' \right] = 0 \quad (9)$$

Subject to the boundary conditions

$$f = 0, \quad f' = 1 + \beta f'', \quad \theta = 1 - \frac{\varepsilon}{4} \quad \text{at } \eta = 0 \quad \text{and} \quad f' \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \quad (10)$$

where, $\text{Pr} = (\mu c_p)_f / k_f$ is the Prandtl number of the base fluid, $\beta = \gamma \sqrt{c / \nu_f}$ is the dimensionless slip parameter, $Ec = u_w^2 / (c_p)_f (T_w - T_\infty)$ is the viscous dissipation parameter (Eckert number), $M = \sigma B_0^2 / c \rho_f$ is the magnetic parameter, $\varepsilon = 4A / n$ is the thermal stratification parameter and $\delta = Q_0 / c (\rho c_p)_f$ heat source / sink parameter.

The important physical quantities of interest are the local skin friction coefficient C_f and local Nusselt number Nu are defined as:

$$C_f = \frac{\tau_{wx}}{\rho_f u_w^2}, \quad Nu_x = \frac{q_w x}{k_f \Delta T} \quad (11)$$

where τ_{wx} , q_w are the surface shear stress along x -direction and wall heat flux which are given by

$$C_f \text{Re}_x^{1/2} = \frac{f''(0)}{(1-\phi)^{2.5}}; \quad \frac{Nu}{Ra_x^{1/2}} = -\frac{k_{nf}}{k_f} \theta'(0) \quad (12)$$

Here $\text{Re}_x = u_w x / \nu_f$ is the local Reynolds number. Effect of the various parameters involved in the investigation on these coefficients is discussed in the following section.

Results and Discussion

The systems of equations (8) and (9) subjects to boundary conditions (10) are transformed into initial value problems by applying Runge-Kutta-Fehlberg in conjunction to shooting methods. Due to no closed form solution to the transformed system, the equations are solved numerically for dimensionless velocity and temperature with the parameter values, $\text{Pr} = 6.2$, $M = 1.0$, $\delta = 0.5$, $\varepsilon = 0.5$ and $Ec = 0.001$ correspond to physically property of Newtonian nanofluid. In addition, the thermophysical properties for different base fluids and CNTs are given in Table 1.

Table 1: Thermophysical properties of different base fluids and CNTs, Khan et al. [7]

Physical properties	Base fluids			Nanoparticles	
	Water	Ethylene glycol	Engine oil	SWCNT	MWCNT
$\rho \text{ (kg / m}^3\text{)}$	997	1115	884	2600	1600
$c_p \text{ (J / kg K)}$	4179	2430	1910	425	796
$k \text{ (W / m K)}$	0.613	0.253	0.144	6600	3000
Pr	6.2	203.63	6450		

In order to validate our computation methods, we have compared the results of $f''(0)$ and the graph profile of present works with those of Rizwan et al. [16] as shown in Table 2 and Figure 2, respectively. It is found that they are in excellent agreement. Moreover, by increasing the number of decimals, the present results are expected to give more accurate than Rizwan et al. [16].

Table 2: Comparison $f''(0)$ with Rizwan et al. [16]

CNT	ϕ	$f''(0)$ at $\beta = 0.5$			
		Rizwan et al. [19]		Present work	
		$M = 0$	$M = 0.5$	$M = 0$	$M = 0.5$
SWCNT	0.0	0.5912	0.6495	0.591178452	0.649493428
	0.1	0.6680	0.7242	0.667965268	0.724248769
	0.2	0.7504	0.8051	0.750374192	0.805072871
MWCNT	0.0	0.5912	0.6495	0.591194527	0.649522965
	0.1	0.6508	0.7106	0.650793523	0.710573241
	0.2	0.7180	0.7795	0.717956234	0.779531892

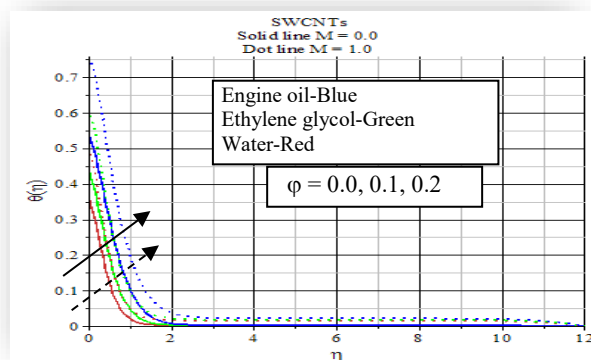


Figure 2: Effects of nanoparticle volume fraction ϕ on temperature profile.

Now, the values of skin friction and rate of heat transfer under effects of magnetic field in the present of thermal stratification is shown in Table 3. It is observed that the absolute values of skin friction increases and the rate of heat transfer decreases with an increase of magnetic strength M .

Table 3: Skin friction and heat transfer rate for different values of M in SWCNT-water.

ε	M	$f''(0)$	$-\theta'(0)$
0.0	0.1	-1.72794282346581	-1.249134833522948
	0.5	-1.92753992015380	-1.220159051139260
	1.0	-2.22603002730154	-1.177754014142869
0.5	0.1	-1.72794282359489	-1.604904554293732
	0.5	-1.94815441748450	-1.564462929698237
	1.0	-2.20344591160449	-1.515270816385395

Figure 3 shows the dimensionless velocity and temperature profiles for different base fluids in the presence of SWCNTs, MWCNTs and magnetic field. Moreover, it can be observed that although the base fluids did not provide a velocity difference but the SWCNT have dominant contribution in the nanofluid flow and in the temperature phenomenon. It is interesting to note that the fluid motion and the temperature distribution are independent of magnetic effects in the presence of SWCNTs and MWCNTs whereas the temperature profiles thickness of the SWCNTs is higher than that of MWCNTs. In the presence of both water based SWCNTs and MWCNTs, it is important to note that the velocity of the fluid, firstly, increases and then decreases whereas the temperature of the fluid decreases with an increase of the water, ethylene glycol and engine oil. For heat transfer characteristics mechanism, interesting result is the large distortion of the temperature field caused by Ethylene glycol and Engine oil. Negative value of the temperature profile in the outer boundary region for Ethylene glycol and Engine oil are due to the combined effects of the strength of viscosity and density of engine-oil-based CNTs.

Figure 4 shows temperature distribution profiles for different values of thermal stratification parameter ε in the presence of water based SWCNTs and MWCNTs with or without magnetic field. In both the cases, it is noticed that the temperature decelerates with increase of the strength of thermal stratification ε because the thermal stratification effect is taken into consideration in an opposing flow along the plate. The effective temperature difference between the plate and the ambient fluid will decrease. This is due to the fact that the thermal stratification parameter ε is directly proportional to the heat transfer coefficient and the dynamic viscosity of the water based SWCNTs and MWCNTs. For thermophysical characteristics mechanism, the large distortion of the temperature field occurs when $0.1 < \varepsilon < 1$. It is also interesting to note that the negative value of the temperature profile at the outer boundary region for $\varepsilon = 0.5$ & 0.9 because the water based SWCNTs and MWCNTs has higher thermal conductivities than the base fluid. It is observed that

the temperature profiles for water based SWCNTs and MWCNTs are very similar to each other with or without magnetic field.

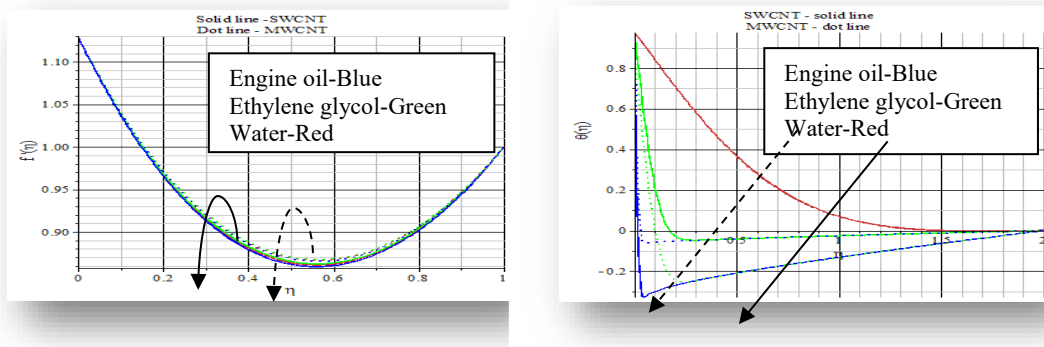


Figure 3: Velocity and temperature profiles for different base fluids in the presence of SWCNTs and MWCNTs with $M = 1.0$.

The effect of nanoparticle volume fraction ϕ on the temperature profile in the presence of water based SWCNTs and MWCNTs with or without magnetic field are illustrated in Figure 5. It is clear that as the nanoparticle volume fraction increases, the nanofluid temperature increases and tends asymptotically to zero. In fact, higher values of thermal conductivity are accompanied by higher values of thermal diffusivity. The high value of thermal diffusivity causes a drop in the temperature gradients and accordingly increases the boundary layer thickness. In convective heat transfer of water based SWCNTs and MWCNTs, the heat transfer depends not only on the thermal conductivity but also on other properties, such as the specific heat, density and dynamic viscosity of a nanofluid.

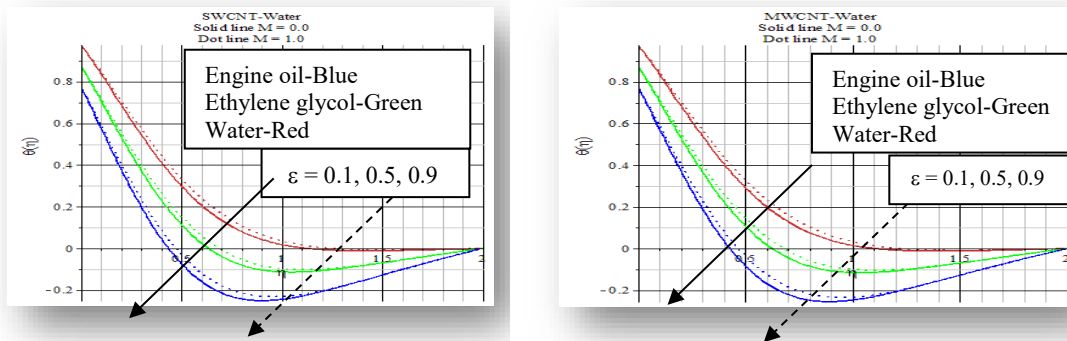


Figure 4: Temperature profile for various value of thermal stratification parameter ε .

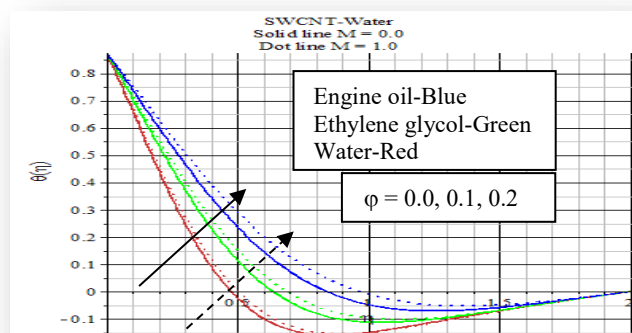


Figure 5: Temperature profile for various value of nanoparticle volume fraction ϕ .

Conclusions

In the present investigation, we have analyzed the performance of CNTs nanofluids on MHD flows over a stretching surface under influence of thermal stratification. It is worthwhile to provide the comparison among the base fluids (water, engine oil and ethylene glycol) in the presence of SWCNTs and MWCNTs. Moreover, effects for various values of existing parameters are discussed for velocity and temperature. The main results of present analysis are listed below:

1. The temperature profile is observed in the outer boundary region for $\varepsilon = 0.5$ & 0.9 , since the SWCNTs and MWCNTs nanofluid have higher thermal conductivities than the base fluid.
2. The sensitivity of thermal boundary layer thickness of CNTs nanofluids is related to the size and shape of the particles of SWCNTs and MWCNTs.
3. In the presence magnetic effect ($M = 1.0$), it is observed that the thermal boundary layer thickness for oil base CNTs nanofluids is smallest compared to that of water and ethylene glycol base CNTs nanofluids.
4. The SWCNTs and MWCNTs nanofluids with an engine oil base fluid shows the highest thermal conductivity in comparison to ethylene glycol and water, respectively.

CNTs nanofluids have attracted much attention for researchers because of their remarkable properties. Indeed, it was proved that the CNTs nanofluids have a high thermal and electrical conductivity. Therefore, CNTs can be a good option to disperse in the common working fluids to enhance the heat transfer.

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